WAVES AND OSCILLATIONS

Def. **2. periodic motion**

Def. **3. Oscillatory Motion**

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OSCILLATION AND WAVES

1. Types of waves
- 1. Non-repetitive motion — Def.
- 2. Periodic motion — Def.
- 3. Repetitive and periodic motion — Def.

Def. **4. Harmonic Oscillation**

Def. **5. Non-Harmonic Oscillations**

Def. **8. Simple Harmonic Motion on other page**

Def. **8. Phase**
- Difference between Phase and Phase difference

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10. Types of equilibrium
- 1. Neutral Eqm — Def.
- 3. Unstable Eqm — Def.
- 2. Stable Eqm — Def.

6. Some definitions
- Def. 2. Frequency
- Def. 3. Angular Frequency
- Def. 4. Displacement
- Def. 1. Time period

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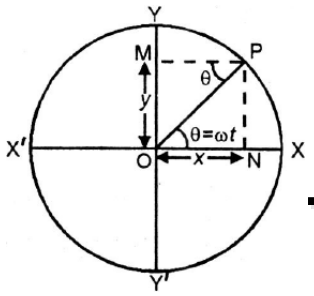
Def. **7. Periodic functions**
- Proof of ω =2π ν

Def. **11. Free Forced and resonant oscillations**
1. Condition of Resonance
2. Amplitude when damping is small or large
3. What is coupled osccilations?

WAVES AND OSCILLATIONS

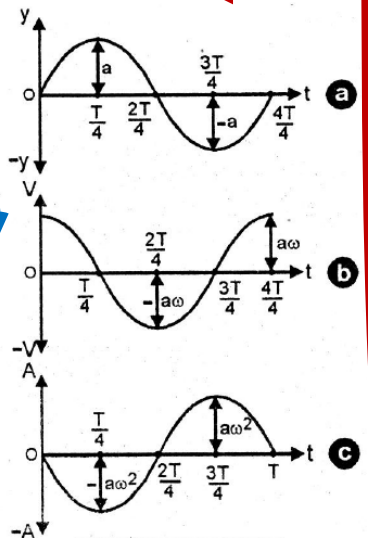


1. Geometrical interpretation from circular motion

SHM

2. Characteristics
- 1. Displacement
  - Meaning of $y=aSin(\omega t+\varphi)$
  - Direction of diaplacement
- 2. Amplitude
  - **Meaning??**
- 3. Velocity
  - **Proof for** $V=\omega\sqrt{a^2-y^2}$
- 4. Acceleration
  - **Proof for** $A=-\omega^2 y$
- 5. Time period
  - **Proof for** Time period, $T=2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}}$

3. Graphs of displacement, velocity and acceleration



4. Total energy in SHM
- 1. Potential energy
- 2. Kinetic energy
- 3. Total energy

1. Proof of formulaes of PE,KE and Total Energy
2. total energy at any istant remains constant

5. Simple Pendulum
6. oscillations of liquid
7. oscillations of floating cylinder
8. oscillations of loaded spring

1. Proof of time period formulas
2. Limitations of simple pendulum formula
3. effective gravity of pendulum when it is immersed in liquid
4. what is second pendulum?
5. g in lift when moving upward or downward with acceleration
6. g in a cart moving with acceleration a

- 1. Time period in horizontal
- 2. Time period in vertical
- 3. Total K for series and parallel combination ?

9. Undamped and damped SHM → Meaning and proof??

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FOR NUMERICAL PRACTICE

IMPORTANT FORMULAE

$$v = \omega\sqrt{A^2 - y^2}$$

$$\nu = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$v_{max} = \omega A$$

$$a_{max} = \omega^2 A$$

W = Increase in P.E. of the spring

$$= \tfrac{1}{2} k (x_2^2 - x_1^2)$$

Total energy of SHM, $E = \frac{1}{2} m\omega^2 A^2$

Kinetic energy of SHM, $E_k = \frac{1}{2} m\omega^2 (A^2 - y^2)$

$$E_p = \frac{1}{2} m\omega^2 y^2$$

Error Calculation

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$\log T = \log 2\pi + \frac{1}{2}\log l - \frac{1}{2}\log g$$

$$\frac{dT}{T} = \frac{1}{2}\frac{dl}{l}$$

Assuming Equations

2. When time measured from extreme position

$$y = A\text{Sin}\omega t$$

1. When time measured from mean position

$$y = A\text{Cos}\omega t$$

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Special Cases and concepts

1. Block in contact of piston doing SHM vertically if $a_{max} \leq g$
2. Person standing on platform measuring weight then

   a. Max. weight = when platform tends to move down as weight and max acceleration of platform in same direction

   b. Min weight= when platform tends to move up as weight and max acceleration of platform in opposite directions
3. If potential energy is not possible to calculate directly, then PE=TE-KE
4. If m_1 and m_2 masses are attached to spring and m_1 is removed suddenly then time period of spring oscillation will be determined by left mass with spring i.e. m_2. Also if y_1 is extension, when both masses are hanging and y_2 is extension when m_2 is hanging only. Then amplitude will be $=y_1-y_2$
5. For a spring in rotation:- $kx=m(r+x)\omega^2$
6. When the mass is hanging between two springs, then springs behave like they are in parallel.
7. Pendulum watch cannot be used in spacecraft as g=0 so T=∞

Given equation is periodic or not?

$$\text{Sin}\omega t \rightarrow \frac{2\pi}{\omega}$$

$$\text{Sin}\omega t + \text{Cos}\omega t = \sqrt{2}\text{Sin}\left(\omega t + \frac{\pi}{4}\right) \rightarrow \frac{2\pi}{\omega}$$

$$\text{Sin}\omega t + \text{Cos}2\omega t + \text{Sin}4\omega t \rightarrow T \text{ and } \frac{T}{2} \text{ and } \frac{T}{4} \rightarrow T = \frac{2\pi}{\omega}$$

Deciphering Equations

Finding Amplitude, angular frequency, frequency, time period, initial phase, particle displacement at t=2 sec, Initial velocity, Acceleration after t sec. for equation X=10Sin(20t+0.5)

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Comparing with general equation $x = A\text{Sin}(\omega t + \phi)$

Amplitude $= 10$

Angular Freq. $\omega = 20$

Frequency $\nu = \frac{\omega}{2\pi}$

Time period $T = \frac{1}{\nu}$

Inital phase $\phi = 0.5$

particle displacement = "put $t = 2$ in given equation"

Velocity $v = A\omega\text{Cos}(\omega t + \phi)$

Acceleration $a = -A\omega^2\text{Sin}(\omega t + \phi)$

Completing equation from given data

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GIVEN:- A=25; T=3;
FIND:- Minimum time required to move between two points 12.5 on either side of mean position.

GIVEN:- pendulum motion; T=2sec; Amplitude=5cm; Starting position at max. displacement
FIND:- equation

Assume equation as $y = A\text{Sin}(\omega t + \phi)$

$$\omega = \frac{2\pi}{T} = \pi$$

$$y = 5\text{Sin}(\pi t + \phi)$$

At time $t = 0$, displacement $y = 5$ cm. Therefore,

$$5 = 5\sin(\pi \times 0 + \phi_0)$$

$$\sin\phi_0 = 1 \quad \therefore \quad \phi_0 = \pi/2$$

$$y = 5\sin\left(\pi t + \frac{\pi}{2}\right) = 5\cos\pi t.$$

Equation $y = 25\text{Sin}\frac{2\pi}{3}t$

$$12.5 = 25\sin\frac{2\pi}{3}t$$

$$\sin\frac{2\pi}{3}t = \frac{12.5}{25} = \frac{1}{2}$$

$$\frac{2\pi t}{3} = \frac{\pi}{6} \quad \text{or} \quad t = \frac{1}{4}\text{ s}$$

$$2t = 2 \times \frac{1}{4} = \frac{1}{2}\text{ s} = 0.5\text{ s}.$$

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Wave Motion

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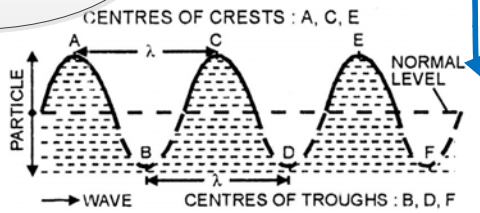
1. Wave motion
   - 1. Energy and momentum transfer
   - 2. no transportation of medium-cork example
   - 3. two properties
     - 1. Inertia- particle does not stop at mean position
     - 2. Elasticity-develop restoring force for max. displacement
   - 4.Crest and trough meaning
   - 5. Mechanical waves- waves in medium

As wave velocity is determined by elastic and inertial properties of medium. So it is constant for a medium and frequency is determined by source so wavelength changes to keep both constant.

2. Types of waves
   - i) **Mechanical waves** Need of material medium
   - ii) **Elecromagnetic waves** -no material medium needed -light speed
   - iii) **Matter waves** -associated with sub atomic particles



3.. Types of mechanical wave motion
   - i) Transverse wave motion
     - Individual particles SHM perpendicular to wave motion
     - need of characteristics of material medium
       - 1. Enough elasticity to bring back particle to mean position
       - 2. Enough Inertia to pass on energy by shooting
       - 3. Minimum friction for minimum loss of energy
       - 4. uniform density of medium
   - ii) Longitudinal wave motion
     - 1. SHM of particles along the direction of waves
     - 2. Compression(High density) and rarefactions(low density)
     - 3. Transmission through solids liquids and gas

Wave will be transverse if medium can sustain shearing strain like solids and longitudinal, if medium can sustain compressive strain

General Equation to solve wave

$$y = r\cos\left[\frac{2\pi t}{T} - \frac{2\pi x}{\lambda}\right]$$

Relation between particle and wave velocity

$\frac{dy}{dx}$ = slope of y wrt x

$\frac{dy}{dt}$ = particle velocity

$\frac{dx}{dt} = \frac{\frac{dy}{dt}}{\frac{dy}{dx}}$ = wave velocity

So ultimately we get

$$u(x,t) = -v\frac{d}{dx}\{y(x,t)\}$$

$\frac{d^2y}{dx^2} = \frac{1}{v^2}\frac{d^2y}{dt^2}$

Condition for being wave

$$y(x,t) = r\sin(\omega t \pm kx + \phi_0)$$

$\omega = \frac{2\pi}{T} = \frac{d\phi}{dt}$

$k = \frac{2\pi}{\lambda} = -d\phi/dx$

$\phi(x,t) = \frac{2\pi}{\lambda}(vt - x) + \phi_0$

6. Harmonic Wave Function
   - Check the below function for time period T and wavelength λ
   - Meaning of following

$$y(x,t) = r\sin\left[\frac{2\pi}{\lambda}(vt - x) + \phi_0\right]$$

5. Periodic Wave Function
   - Meaning of following
   - Derivation

$y[(x + n\lambda);t] = y[x,t]$,

$y[x,(t + mT)] = y[x,t]$

4. Wave Function
   - Why distance and time both are important?
   - Derivation of y=f(x±vt); When + or -?

Wave Motion

If a string is vibrating so as to produce nth harmonic, its frequency will be nv. Number of loops = n. Number of antinodes =n. Number of nodes = (n + 1).

STRING LAWS
1. Law of length
2. Law of tension
3. Law of mass
4. Law of diameter
5. Law of density

FOR STRING
1. prove Y(x,t)=-(2rSinkx)Cosωt
2. For Node:- prove λ =2L/n
3. first overtone is second hamonic.
4. General formula for frequency of vibration.

CHARACTERISTCS
1. Disturbance to a particular wave
2. No disturbance movement
3. No flow of energy
4. distance between two consecutive nodes or antinodes is λ/2(Prove!!)

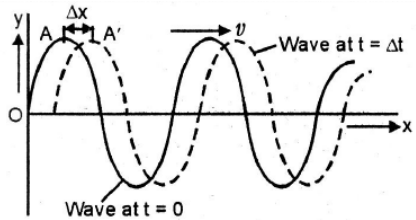
Standing Waves
1. Two exactly similar waves travelling in opposite directions
2. No propagation of waves or energy
3. Nodes and antinodes concept
4. Longitudinal and Transverse waves

Principle of superposition of waves
Show superposition on following waves

$$y_1(x,t) = r\sin(\omega t - kx)$$

$$y_2(x,t) = r\sin(\omega t - kx + \phi)$$

REFLECTION OF WAVES
1. hard and plane wooden surface for sound
2. follow laws of reflection and mirror formula
3. Reflection of waves at rigid boundary(denser medium)→ reflect at 180^0 phase difference to cancel the displacement of support.
4. The reflection at open boundary occurs without any phase change.
5. Wave equations from rigid and open boundary



13. Sound waves
1. Change in density --> Change in pressure --> restoring force proptional to disturbance develops
2. Audible, Ultrasonic and Infrasonic vibrations
3. supersonic speed
4. Wave form displacement → Proof of $v = \frac{\omega}{k} = \nu\lambda$

Speed of transverse wave on stretched string
Proof of formula by dimensional Analysis
What is m??
$$\mathbf{v = \sqrt{\frac{T}{m}}}$$

Speed of longitudinal waves
2. In Liquids $\boldsymbol{v = \sqrt{\frac{B}{\rho}}}$
1. In Gases $\boldsymbol{v = \sqrt{\frac{p}{\rho}}}$
LAPLACE CORRECTION $\boldsymbol{v = \sqrt{\frac{\gamma p}{\rho}}}$
Calculation of B from Newton's and laplace both

Factors Affecting Velocity of Sound
1. Effect of density- Inverse
2. Effect of pressure- no effect
3. Effect of Temperature-Direct Prop.
Temperature Coeff. derivation
4. Effect of Humidity- decreases?
5. Effect of Wind velocity-vector addition